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Dielectric function of a magnetoactive degenerate plasma at relativistic temperature*

By P. Misra

Departmentfof Physics, S.C.S. College, Puri

AND

K. C. Roy

Department of Physics, Anugul Science College, Anugul

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Using relativistic Boltzmann-Vlassov equation, expression for the frequency and wave-number dependent dielectric function of a degenerate electron plasma is derived in presence of a steady magnetic field, which is valid at temperatures satisfying the condition $mc^2 \ll KT$. The result is compared with those obtained for a plasma under different physical conditions.

Introduction

In this paper the dielectric constant of a degenerate plasma in presence of a uniform magnetic field is derived in a manner similar to our earlier work (Misra *et al* 1970). The expression is valid in the extreme relativistic limit, *i.e.* $KT \gg mc^2$, and for the case when the imposed steady magnetic field is parallel to the wave-number vector.

Dielectric function of a magnetoactive relativistic plasma

The linearised relativistic Boltzmann-Vlassov equation in presence of an external magnetic field can be written as

$$\frac{\partial f(\vec{r}, \vec{p}, t)}{\partial t} + \frac{c\vec{p}}{(p^2+m^2c^2)^{\frac{1}{2}}} \cdot \nabla_r f_1(\vec{r}, \vec{p}, t)$$

$$= n_0 e\vec{E}(\vec{r}, t) \cdot \vec{\nabla}_p f_0(p) - \frac{e(\vec{p} \times \vec{B})}{(p^2+m^2c^2)^{\frac{1}{2}}} \cdot \vec{\nabla}_p f_1(\vec{r}, \vec{p}, t) \quad \ldots \quad (1)$$

whose space-time Fourier transform, in a polar co-ordinate system (p, θ, ϕ) for $\vec{p}$ with $\vec{B}$ and $\vec{k}$ along z axis, gives

$$f_1(\vec{k}, \vec{p}, \omega) = -\frac{n_0 A}{B} (p^2+m^2c^2)^{\frac{1}{2}} \frac{\partial f_0}{\partial p} \times$$

$$\left((E_1 \sin\theta \frac{A \sin\phi - \cos\phi}{A^2+1} - E_2 \sin\theta \frac{A \cos\phi + \sin\phi}{A^2+1} - E_3 \cos\theta \right) \quad \ldots \quad (2)$$

*A part of the work was done at the Ravenshaw College, Cuttack where

$$A = \frac{eB}{i[cpk \cos\theta - \omega(p^2+m^2c^2)^{\frac{1}{2}}]}$$

When equation (2) is substituted in the wavenumber and frequency dependent current density (Gartenhaus 1964)

$$J_\alpha(\vec{k}, \omega) = -ec \int \vec{dp}\, \frac{p_\alpha f_1(\vec{k}, \omega, \vec{p})}{(p^2+m^2c^2)^{\frac{1}{2}}}$$

$$\equiv K_{\alpha\beta}(\vec{k}, \omega) E_\beta(\vec{k}, \omega) \qquad \ldots (3)$$

we get the following expressions for the Response functions

$$K_{11} = K_{22} = -\frac{\pi n_0 ec}{B} \int_9^\infty \int_0^\pi \frac{Ap^3 \sin^3\theta dp d\theta}{A^2+1} \frac{\partial f_0}{\partial p} \qquad \ldots (4)$$

$$K_{12} = -K_{21} = -\frac{\pi n_0 ec}{B} \int_0^\infty \int_0^\pi \frac{A^2 p^3 \sin^3\theta d\theta dp}{A^2+1} \frac{\partial f_0}{\partial p} \qquad \ldots (5)$$

$$K_{33} = -\frac{2\pi n_0 ec}{B} \int_0^\infty \int_0^\pi A \left(\frac{\partial f_0}{\partial p}\right) p^3 \cos^2\theta \sin\theta\, dp d\theta \qquad \ldots (6)$$

After some elementary calculations equations (4), (5) and (6) become,

$$K_{tr}^{(\pm)} = -\frac{i\omega_0^2 mc}{2} \int_0^\infty \int_0^1 \frac{dp dx p^3[\omega(p^2+m^2c^2)^{\frac{1}{2}} \pm eB](1-x^2)}{[\omega(p^2+m^2c^2)^{\frac{1}{2}} + eB]^2 - c^2p^2k^2x^2} \times \frac{\partial f_0}{\partial p} \qquad (7)$$

and

$$K_l = -iw_0^2 \omega mc \int_0^\infty \int_0^1 \frac{dp dx p^3 (p^2+m^2c^2)^{\frac{1}{2}} x^2}{\omega^2(p^2+m^2c^2) - c^2p^2k^2x^2} \frac{\partial f_0}{\partial p} \qquad \ldots (8)$$

where $K_{tr}^{(\pm)} = K_{11} \pm K_{12}$ and $K_l = K_{33}$. From the equation (8) it is clear that in a relativistic plasma the longitudinal mode is also not affected by the magnetic field. Using the equation (7) in the relation,

$$\epsilon_{tr} = 1 - \frac{4\pi i K_{tr}(k, \omega)}{}$$

we can obtain the expression for the dielectric function.

EVALUATION OF DIELECTRIC CONSTANT FOR RELATIVISTIC FERMI DISTRIBUTION

Using the value of equilibrium relativistic Fermi distribution, f_0 in equation (7) we have

$$\operatorname{Im} K_{tr}^{(\pm)} = \frac{\omega_0^2 m}{k^2 KT n_0 h^3 \Lambda} \times$$

$$\int_0^\infty \frac{p^2 dp[\omega(p^2+m^2c^2)^{\frac{1}{2}} \pm eB] \exp\left[\frac{c}{KT}(p^2+m^2c^2)^{\frac{1}{2}}\right]}{\left[\frac{1}{\Lambda}\exp\left\{\frac{c}{KT}(p^2+m^2c^2)^{\frac{1}{2}}\right\}+1\right]^2 (p^2+m^2c^2)^{\frac{1}{2}}} \times$$

$$\left[1+\frac{c^2k^2p^2-[\omega(p^2+m^2c^2)^{\frac{1}{2}} \pm eB]^2}{2ckp[\omega(p^2+m^2c^2)^{\frac{1}{2}} \mp eB]} \ln\left|\frac{\omega(p^2+m^2c^2)^{\frac{1}{2}} \pm eB+cpk}{\omega(p^2+m^2c^2)^{\frac{1}{2}} \pm eB-cpk}\right|\right] \quad \ldots \quad (9)$$

where $n_0 f_0(p) = \frac{2}{h^3}\left[\frac{1}{\Lambda}\exp\left\{\frac{c}{KT}(p^2+m^2c^2)^{\frac{1}{2}}\right\}+1\right]^{-1}$

and $$\frac{1}{\Lambda} = \exp\left[-\left(\nu+\frac{mc^2}{KT}\right)\right]$$

It can easily be shown that for $c = \infty$ and $B = 0$ equation (9) reduces to the corresponding non-relativistic equation given by Misra *et al* (1962) in equation (12)

The integrals occurring in equation (9) cannot be analytically carried out. However, for temperatures, $KT >> mc^2$, approximate analytical forms can be obtained by using Sommerfeld's method (1928) of integration as was done in our earlier work (Misra *et al* 1970), and we give the final result

$$\epsilon_{tr} = 1-\frac{3\omega_0^2}{2ck^2v_0^3}\left[\frac{v_0(\omega v_0 \pm \Omega c)}{\omega}\right.$$

$$\left.+\ c^2k^2v_0^2-\frac{(\omega v_0 \pm \Omega c)^2}{2ck\omega} \ln\left|\frac{\omega v_0 \pm \Omega c+ckv_0}{\omega v_0 \pm \Omega c-ckv_0}\right|\right]$$

$$-\frac{\pi^2\omega_0^2K^2T^2}{2k^2v_0^3m^2c^3}\left[1+\frac{c^2k^2-\omega^2}{2ck\omega} \ln\left|\frac{\omega v_0 \pm c\Omega+ckv_0}{\omega v_0 \pm \Omega c-ckv_0}\right|\right.$$

$$\left.+\frac{c(\omega c \pm \Omega v_0)[(c^2k^2-\omega^2)v_0 \pm \Omega c\omega]}{\omega[(\omega v_0 \pm \Omega c)^2-c^2k^2v_0^2]v_0}\right] \quad \ldots \quad (10)$$

where we have taken $\Omega = \frac{eB}{mc}$ amd $c \ll v_0 = \left(\frac{3n_0}{8\pi}\right)\left(\frac{h}{m}\right)$. In the limit of vanishing B, the expression for ϵ_{tr} coincides with that obtained for the magnetic field free case (Misra *et al* 1970). Further if we put $c = v_0$ in equation (10), the temperature independent term becomes identically equal to that obtained for non-relativistic case given by Misra *et al* (1969) in equation (5).

Discussion of the result

Propagation of electromagnetic waves can be analysed with the aid of equation (10). As the expression is complicated and lengthy we discuss only the nature of propagation in certain limiting cases.

Case I $$\frac{v_0 ck}{\omega v_0 \pm \Omega c} < 1$$

Case II $$\frac{\omega v_0 \pm \Omega c}{v_0 ck} < 1$$

The first case is satisfied for $ck < \omega$ and $\omega v_0 > \Omega c$. With this approximation and simple calculation, we get

$$\epsilon_{tr} = 1 - \frac{\omega_0^2 c}{\omega(\omega v_0 \pm \Omega c)}\left[1 + \frac{\pi^2 K^2 T^2}{2m^2c^2v_0^2}\left(1 + \frac{\omega^2}{k^2 v_0^2}\right)\right]$$

$$\simeq 1 - \frac{\omega_0^2 c}{\omega(\omega v_0 \pm \Omega c)}$$

We see from this equation that waves with frequencies for which

$$\omega(\omega v_0 \pm \Omega c) < \omega_0^2 c$$

cannot propagate through the relativistic degenerate plasma. Whereas the corresponding condition for a non-relativistic degenerate plasma is given by $\omega(\omega \pm \Omega) < \omega_0^2$, (Misra *et al* 1969).

The second case is satisfied for $\omega v_0 \sim \Omega c$ as well as for ω, for which $ck > \omega$. Under this approximation we get

$$\epsilon_{tr} = 1 - \frac{3\omega_0^2 c(\omega v_0 \pm \Omega c)}{c^2 k^2 v_0^2 \omega}$$

$$- \frac{\pi^2 \omega_0^2 K^2 T^2}{2k^2 v_0^3 m^2 c^3}\left[1 + \frac{(c^2k^2 - \omega^2)(\omega v_0 \pm \Omega c)}{c^2 k^2 \omega v_0}\right]$$

Since the term containing temperature is small we can write

$$\epsilon_{tr}(\epsilon_{tr} - 1) \simeq -\frac{3\omega_0^2 c(\omega v_0 \pm \Omega c)}{\omega^3 v_0^2}$$

where $$\epsilon_{tr} \simeq \frac{ck}{\omega}$$

From the above equation it is clear that for the extraordinary wave, when

$$\omega v_0 = \Omega c, \quad \epsilon_{tr} = 1$$

$$\omega v_0 > \Omega c, \quad \epsilon_{tr} < 1$$

and $$\omega v_0 < \Omega c, \quad \epsilon_{tr} > 1$$

The equation (12) is to be compared with the corresponding equation for the non-relativistic plasma, which is quoted below from Misra *et al* (1969),

$$\epsilon_{tr}(\epsilon_{tr}-1) \simeq \frac{3\omega_0^2 c^2(\omega \pm \Omega)}{\omega^3 v_t^2}$$

We then conclude that the imposition of a strong magnetic field makes plasma transparent to very low frequency electromagnetic waves which in the absence of the field cannot be propagated. The same conclusion can be made from equation (12) with the difference that magnetic field necessary to make the plasma transparent will be v_0/c times greater than that required for the non-relativistic case.

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